

LECTURE: 5-5 THE SUBSTITUTION RULE (PART 2)

Example 1: Evaluate the following indefinite integrals.

(a) $\int \frac{\cos \theta}{\sin^2 \theta} d\theta$

(b) $\int \tan x dx$

Example 2: Evaluate the following indefinite integrals.

(a) $\int (1 + \tan x)^5 \sec^2 x dx$

(b) $\int \frac{\cos(\pi/x)}{x^2} dx$

Example 3: Evaluate $\int \frac{5+x}{1+x^2} dx$.

Sometimes when you do substitution you also end up solving for your variable in the substitution. For example:

Example 4: Evaluate $\int x^5 \sqrt{x^3 + 1} dx$.

Example 5: Evaluate $\int x \sqrt{x + 2} dx$

Definite Integrals

The Substitution Rule for Definite Integrals: If g' is continuous on $[a, b]$ and f is continuous on the range of $u = g(x)$, then

$$\int_a^b f(g(x))g'(x) = \int_{g(a)}^{g(b)} f(u)du$$

Example 6: Evaluate $\int_0^{\pi/2} \sin^3 x \cos x dx$ two ways:

a) going back to x 's

b) using substitution

Example 7: Evaluate the following definite integrals.

a) $\int_e^{e^3} \frac{1}{x(\ln x)^2} dx$

b) $\int_1^2 x\sqrt{x-1} dx$

Example 8: Evaluate the following definite integrals.

a) $\int_0^1 2^z \sin(2^z) dz$

b) $\int_0^2 \frac{x}{x^2+4} dx$

Symmetry

- A function f is even if _____. Even functions are symmetric about the _____.
- A function f is odd if _____. Odd functions are symmetric about the _____.

Integrals of Even/Odd Functions: Suppose a function $f(x)$ is (blank) on $[-a, a]$. Then,

(a) (even) $\int_{-a}^a f(x)dx$

(b) (odd) $\int_{-a}^a f(x)dx$

Example 9: Evaluate the following definite integrals.

(a) $\int_{-2}^2 (x^2 + 1)dx$

(b) $\int_{-1}^1 \frac{\tan x}{1 + x^2} dx$

Example 10: If f is continuous and $\int_0^9 f(x)dx = 4$, find $\int_0^3 xf(x^2)dx$.